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Friction factor and Nusselt number in flat tubes with rounded edges

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This paper is aimed at presenting a fluid flow and heat transfer analysis in flat rectangular ducts with rounded edges. The differential momentum and energy equations are numerically solved by a finite difference technique for a fluid in steady-state, laminar flow with constant wall temperatures. The two-dimensional (2-D) velocity and temperature profiles are so determined, and the friction factors and the Nusselt numbers are predicted for different aspect ratios ($0 \leq \alpha \leq 0.5$) for Newtonian and slug flows. The results show that the differences in friction factor and Nusselt number between round-edged ducts and rectangular ducts increase when the aspect ratio increases. Very accurate results are predicted by simple polynomial expressions.

Keywords: thermohydraulics; temperature profile; round-edged tubes

Introduction

The fluid dynamics and heat transfer behavior of flows through rectangular ducts with rounded edges (or "stadium-shaped") is of special interest because of the wide application in compact heat exchangers. These heat exchangers require tubes with a low hydraulic diameter for a given cross section surface, in order to have a high ratio of heat transfer area to exchanger volume (Taborek, et al. 1983). Flat tubes meet this requirement, and for this reason, they are often used in heat exchanger design. Automotive and truck radiators are so widely used that they afford a typical application of such heat exchangers; the coolant air passes through the radiator, while the hot fluid in the tubes is liquid at comparatively low Reynolds number.

Experience indicates that flat tubes give the optimum heat transfer performance, and the matrices are light, yet adequately strong and not expensive to fabricate (Fraas 1989). Flattened rectangular tubes are the most common, but manufacturing considerations impose tubes with rounded edges.

In the rectangular duct, for laminar flow, the friction factor and the Nusselt number were obtained and described in detail in several papers (Shah and London 1978; Hartnett and Kostic 1989). Although information is scarce for the round edged tubes, which represent the flat tubes used in most radiators, only in Kakaç, Shah, and Aung (1987) are stadium-shaped ducts dealt with.

The fRe product and the Nusselt numbers for fluids with constant properties in fully developed laminar flow are constant, independent of Reynolds and Prandtl numbers, but dependent upon the flow passage geometry and thermal boundary conditions. The boundary conditions play a fundamental role for the prediction of the Nusselt numbers. In the literature, three classes of boundary conditions are examined: the T condition (referred to constant wall temperature throughout the channel length), the $H1$ condition (referred to constant wall temperature in the peripheral direction and constant heat transfer rate in the axial direction), and the $H2$ condition (referred to constant heat transfer rate in the peripheral direction as well as in the fluid axial direction).

The aim of this paper is the analysis of the hydrodynamics and heat transfer of flat tubes with rounded edges, for fully developed laminar flow, with $H1$ boundary conditions. The results obtained for this kind of duct are compared with those obtained by the same procedure applied to rectangular ducts having equal encumbrance; i.e., equal cross section length and width, as shown in Figure 1, where the two different sections are superposed. The shorter side 2b of the rectangle is substituted, in the round-edged tube, by a half circumference with radius b.

Governing equations and numerical solution

Friction factor and Nusselt number cannot be treated until velocity and temperature distributions are specified. To reach this goal, consider a steady laminar, hydrodynamically fully developed flow of a Newtonian fluid, in forced convection in a rectangular duct. To simplify the analysis, some assumptions are made.

(1) The physical properties are constant.
(2) The incompressible fluid is flowing in a horizontal tube.
(3) The pressure gradient is constant in the flow direction.
(4) There are no heat sources and no viscous dissipation within the fluid.
(5) According to $H1$ condition, the temperature gradient is constant in the flow direction, and the wall temperature is constant over every cross section.

With these assumptions, the momentum Navier–Stokes equation becomes as follows:

$$\frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{1}$$

with the boundary nonslip condition $u = 0$ at the walls, while the energy equation reads as follows:

$$\rho c u(x, y)\frac{\partial T}{\partial z} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2}$$

with the $H1$ condition $T = T_{\text{wall}}$ on the walls.

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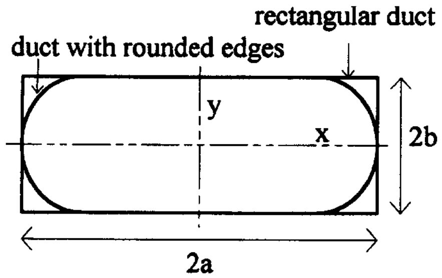

Figure 1 Rectangular and round-edged ducts with equal section length and width

The origin of the Cartesian coordinate system is at the center of the duct. Analytical and numerical solutions of Equation 1 applied to rectangular ducts have been provided (Spiga and Morini 1994; Gao and Hartnett 1993). No solution is available in the literature for ducts with rounded edges.

The finite difference method allows us to represent the momentum and energy partial differential Equations 1 and 2 in algebraic form. A rectangular duct does not require any particular treatment for finite difference method: a 100×100 point grid has been used regardless of the duct aspect ratio (Salvadori and Baron 1961). A modern PC equipped with 80486 processor takes only a few minutes to achieve the final result resorting to a Fortran program based on an iterative solution method. On the other hand, round-edged ducts require a special treatment in order to account for the irregular boundary on the rounded walls (a coarse mesh is shown for simplicity in Figure 2).

Taylor's series second-order expansions about the point i,j give the following results for the local velocity:

$$u_{i,j+1} = u_{i,j} + \Delta x_{\text{edge}} \left.\frac{\partial u}{\partial x}\right|_{i,j} + \Delta^2 x_{\text{edge}} \left.\frac{\partial^2 u}{\partial^2 x}\right|_{i,j} \tag{3}$$

$$u_{i,j-1} = u_{i,j} + \Delta x \left.\frac{\partial u}{\partial x}\right|_{i,j} + \Delta^2 x \left.\frac{\partial^2 u}{\partial^2 x}\right|_{i,j} \tag{4}$$

where the segments Δx and Δx_{edge} are shown in Figure 2.

Assuming the usual approximation:

$$\left.\frac{\partial u}{\partial x}\right|_{i,j} = \frac{u_{i,j+1} - u_{i,j-1}}{\Delta x + \Delta x_{\text{edge}}} \tag{5}$$

the second partial derivative of $u(x, y)$ is obtained by addition of the expressions 3 and 4: it reduces to an ordinary finite difference correlation if $\Delta x_{\text{edge}} = \Delta x$. Analogously, the second derivative with respect to y is easily obtained; the same procedure can be applied to represent the second derivatives of T with respect to x and y. The number of grid points chosen for the analysis of the round-edged duct ranges from 80×160 for $\alpha = 1/2$ to 80×80 for $\alpha \to 0$. The numerical solution to the algebraic system is obtained by an iterative procedure, resorting to the relaxation method, a modified procedure of the Gauss–Seidel technique giving the numerical convergence in shorter times.

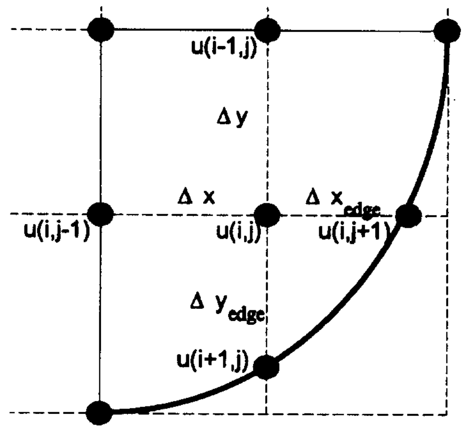

Figure 2 Grid points arrangement on the duct boundary

Results

The solution to the governing equations allows us to deduce the two-dimensional (2-D) velocity and temperature distribution in the rectangular cross section. Figure 3 shows the velocity distribution for rectangular and round-edged ducts, with equal cross section length and width; and $\alpha = 0.2$. Different gray shades stand for different velocity intervals, ranging from the maximum value at the center of the section to 0 at the walls; every color variation indicates 10% variation of u. Rectangular ducts have a wide area with very low velocity in the wall corners. This fact doesn't apply to round-edged ducts. Of course, the numerical results concerning rectangular and round-edged ducts coincide when α tends to 0.

The average velocity is determined through a surface integral over the cross section of the duct. Figure 4 shows the maximum to average velocity ratio in the two ducts; the difference increases with α (3.3% for $\alpha = 0.5$). The aspect ratio is chosen less than 0.5

Notation

a, b	longer and shorter half sides, respectively, of the rectangle, m
c	specific heat, J/kg K
D	equivalent diameter $4ab/a+b$, m
f	friction factor
h	heat transfer coefficient, W/m^{2}K
k	thermal conductivity, W/mk
n	coordinate normal to the wall, m
Nu	Nusselt number, hD/k
p	pressure, Pa
Re	Reynolds number, $\rho WD/\mu$
$T(.)$	fluid temperature, K
T_{bulk}	fluid bulk temperature, K
T_{wall}	wall temperature, K
$u(.)$	fluid velocity, m/s
W	average velocity, m/s
x, y	rectangular coordinates, m
z	axial coordinate in the flow direction, m

Subscripts

rect	rectangular
rounded	round-edged

Greek

α	aspect ratio, $b/a < 1$
Δx	step length in the x direction, m
Δx_{edge}	step length in the x direction near the rounded edge, m
μ	fluid viscosity, Pa s
ρ	fluid density, kg/m^3

Figure 3 Velocity distribution in 1/4 cross section in the rectangular duct and the rounded edges tube

in order to represent flat rectangular tubes.

The knowledge of the velocity distribution allows us to deduce the friction factor as a function of the Reynolds number; as usual it is defined as follows:

$$f = \frac{2D\left|\frac{\partial p}{\partial z}\right|}{\rho W^2} \tag{6}$$

Figure 5 reports the fRe product of both types of tubes having equal encumbrance. Results are reported for different values of the rectangular duct aspect ratio for flattened tubes ($0 \leq \alpha \leq 0.5$).

The temperature distributions for the ducts are shown in Figure 6, for $\alpha = 0.2$. Knowledge of the temperature distribution allows us to calculate the temperature derivatives and the fluid bulk temperature. As usual the local heat transfer coefficient is defined as follows:

$$h_{\text{local}} = \frac{k\left|\frac{\partial T}{\partial n}\right|_{\text{wall}}}{|T_{\text{wall}} - T_{\text{bulk}}|} \tag{7}$$

The rectangular duct corners present very low local heat transfer coefficients because of the flat temperature profile. In fact, in the corner region the color is constant, so the temperature of the fluid does not change significantly. This does not happen for round-edged ducts, which present more regularly distributed local heat transfer coefficients. The heat transfer coefficient h, referred to the whole cross section, is the average local heat transfer coefficient, calculated as the line integral on the wetted perimeter of h_{local} divided by the perimeter length.

Finally, the same problem has been solved for slug flow, which represents the limiting case of a non-Newtonian fluid with a rheological power law whose index goes to zero. The problem is easier, because the solution to Equation 1 is now $u(x, y) = \text{constant}$ and the left-hand side of Equation 2 is constant. In slug flow, the velocity profile is quite flat, and the temperature profile is more uniform, hence the local Nusselt numbers are higher, with respect to the Newtonian flow. In Figure 7 the Nusselt numbers ($\text{Nu} = hD/k$) are reported for slug and Newtonian flows for the two kinds of tubes versus the aspect ratio.

All the numerical results concerning the rectangular duct are in perfect agreement with the results available in the literature, and the numerical solution for $\alpha \to 0$ provides the well-known slab geometry results.

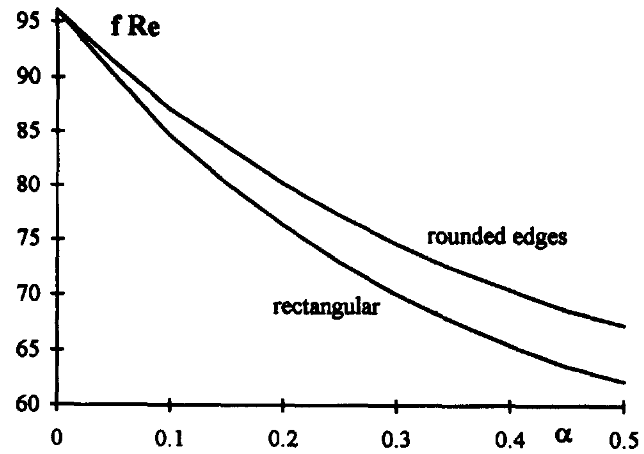

Figure 5 Friction factor-Reynolds number product for rectangular and round-edged ducts

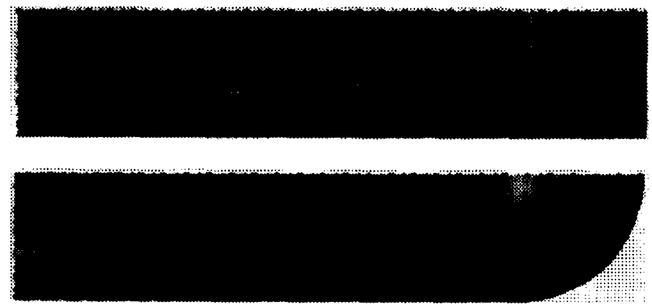

Figure 6 Temperature distribution in 1/4 cross section of the rectangular duct and the rounded edges tube

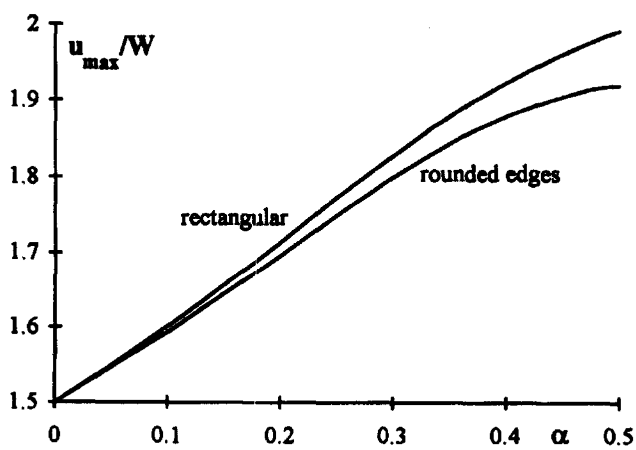

Figure 4 Maximum-to-average velocity ratio in rectangular and rounded-edge ducts

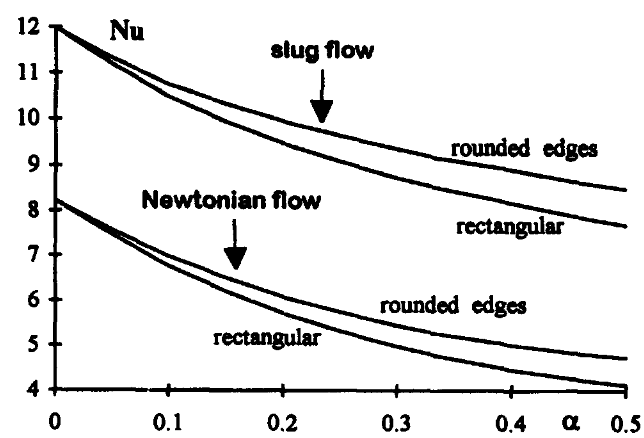

Figure 7 Nusselt number in slug flow and laminar Newtonian flow, for rectangular and round-edged ducts

In conclusion, this analysis puts in evidence higher values for fRe and Nu in the round-edged duct, compared to the rectangular duct. The results plotted in Figures 5 and 7 for laminar Newtonian flow are represented versus the aspect ratio by the polynomial expressions:

$$\begin{cases} \dfrac{(f\mathrm{Re})_{\mathrm{rounded}}}{(f\mathrm{Re})_{\mathrm{rect}}} = 1 + 0.180339\alpha + 2.46175\alpha^2 - 19.3645\alpha^3 \\ \qquad +60.0465\alpha^4 - 85.5526\alpha^5 + 45.9244\alpha^6 \\ \dfrac{\mathrm{Nu}_{\mathrm{rounded}}}{\mathrm{Nu}_{\mathrm{rect}}} = 1 + 0.249365\alpha + 1.51039\alpha^2 - 11.914\alpha^3 \\ \qquad +43.3224\alpha^4 - 74.759\alpha^5 + 48.9664\alpha^6 \end{cases} \tag{8}$$

Agreement between the polynomials (Equation 8) and the numerical solution is within 0.56%. In a similar analysis quoted in Kakaç, Shah, and Aung (1987), the fRe prediction agrees with the numerical results within 3%.

Finally, the Nusselt number in slug flow for round-edged ducts in *H1* conditions is given, with an agreement within 0.40%, by the approximate polynomial:

$$\left(\frac{\mathrm{Nu}_{\mathrm{rounded}}}{\mathrm{Nu}_{\mathrm{rect}}}\right)_{\mathrm{slug}} = 1 + 0.313899\alpha - 2.4771\alpha^2 + 32.8239\alpha^3 - 198.574\alpha^4 + 600.666\alpha^5 - 894.86\alpha^6 + 522.775\alpha^7 \tag{9}$$

Conclusions

The numerical solution to the problem with *H1* boundary conditions in stadium-shaped ducts has been obtained in order to predict friction factors and Nusselt numbers. The main features of the paper consist in the presentation of 2-D velocity and temperature profiles, the fRe and Nu predictions, original for slug flow and improved, with respect to the existing literature, for Newtonian flow.

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